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MATH 116
CHAPTER 2.3

20) We have $a, b, c, m, n \in \mathbb{Z}$ and $a|b$, $a|c$.
Since $a|b$, there is $k \in \mathbb{Z}$ st $ak = b$
Since $a|c$, there is $s \in \mathbb{Z}$ st $as = c$

Then $mb + nc = m(ak) + n(as)$
$= (mk)a + (ns)a$ .... (by commutativity and associativity of '$\cdot$' in $\mathbb{Z}$)
$= (mk + ns)a$ .... (by distributivity '$\cdot$' over '$+$' in $\mathbb{Z}$)

Since $\mathbb{Z}$ is both closed under '$+$' and '$\cdot$', $mk + ns \in \mathbb{Z}$. So there is an integer $(mk+ns)$ st $mb + nc = (mk+ns)a$.
Hence by definition of divisibility, $a | (mb + nc)$.

21) We have $a, b \in \mathbb{Z}$ and $a|b$, $b|a$.
Since $a|b$, there is $k \in \mathbb{Z}$ st $ak = b$.
Since $b|a$, there is $s \in \mathbb{Z}$ st $bs = a$.
Note that if $b = 0$, then $a = 0$. Hence $a = b$.
Assume $b \neq 0$. We ~~have~~ $b = ak = (bs)k$
$= b(sk)$ ...... (associativity of '$\cdot$' in $\mathbb{Z}$)
Then $sk = 1$ because $0 = b - bsk = b(1 - sk)$, and $b \neq 0$.
Since $s|1$, $s \in \mathbb{Z}$ and only divisors of 1 are 1 and -1, s is 1 or -1.
Then, if $s = 1$ then $a = b$ or
if $s = -1$ then $a = -b$.

22) We have $a, b \in \mathbb{Z}$, $a|b$ and $b \neq 0$.
Since $a|b$, there is $k \in \mathbb{Z}$ st $ak = b$. Since $b \neq 0$, $k \neq 0$, $a \neq 0$

Since $ak = b$, $|ak| = |b|$ and $|a||k| = |b|$.
Then $|b| - |a| = |a||k| - |a|$
$= |a|(|k| - 1)$. Since $k \in \mathbb{Z}$, and $k \neq 0$, $|k| \geq 1$
If $|k| = 1$ then $|b| - |a| = 0$ and then $|b| = |a|$.
If $|k| > 1$ then $|a|(|k| - 1) > 0$ because $|a| \in \mathbb{Z}^+$, $|k| - 1 \in \mathbb{Z}^+$.
Hence $|b| - |a| > 0$, that means $|b| > |a|$.