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# MATH 116
## EXERCISES 8.1

3) (f) Let $f(x) = 2x^3 + 7x + 4$, $g(x) = 4x^2 + 4x + 6$ and $h(x) = 6x^2 + 3$ be elements of $\mathbb{Z}_8[x]$.

Then
$$\begin{aligned} f(x) + g(x)h(x) &= (2x^3+7x+4) + (4x^2+4x+6)(6x^2+3) \\ &= (2x^3+7x+4) + (24x^4+24x^3+36x^2+12x^2+12x+18) \\ &= (2x^3+7x+4) + (4x+2) \\ &= 2x^3+11x+6 \\ &= 2x^3+3x-2 \end{aligned}$$

5) (d) Let $R[x]$ be a polynomial ring. Then
$A = \{$the set consisting of the zero polynomial together with all polynomials that have degree 2 or less$\}$ <u>does not</u> form a subring of $R[x]$.
Because, $(x^2+1) \in A$ but $(x^2+1)(x^2+1) = x^4 + 2x^2 + 1 \notin A$

11). Define $\theta : R \longrightarrow R[x]/_I$ where $I = (x)$, by

$\theta(a) = a + I$ for all $a \in R$.

Then $\theta(ab) = ab + I = (a+I)(b+I) = \theta(a)\theta(b)$ for all $a, b \in R$

and $\theta(a+b) = (a+b) + I = (a+I) + (b+I) = \theta(a) + \theta(b)$ " ".

Therefore $\theta$ is a ring homomomorphism.

Claim; $\theta$ is onto;

Let $f(x) + I \in R[x]/I$, and let $a_0$ be the constant term of $f(x)$. Then $f(x) + I = \theta(a_0) = a_0 + I$ because $f(x) - a_0 = x \cdot g(x)$ for some $g(x) \in R[x]$, that is, $f(x) - a_0 \in (x) = I$. Therefore $\theta$ is onto.

Claim: $\theta$ is one to one;

Suppose $\theta(a) = \theta(b)$ for some $a, b \in R$.

Then $a + I = b + I$, hence $a - b \in I = (x)$. Since $a - b$ is a constant, it has no $x$-term in it. Hence $a - b = 0$.

Then $a = b$. Therefore $\theta$ is one-to one.