Math 116, Spring 2008.

5.1.13) Let R be a ring and $a \in R$.

Then $0 \cdot a = (0+0) \cdot a = 0 \cdot a + 0 \cdot a$. Hence

$$0 = 0 \cdot a - 0 \cdot a = 0 \cdot a + 0 \cdot a - 0 \cdot a = 0 \cdot a.$$

15) G is a ring because the defined multiplication is associative as $a(b \cdot c) = 0$ & $(ab) \cdot c = 0$ for all $a, b, c \in R$,

and $a \cdot (b+c) = 0$ and $a \cdot b + a \cdot c = 0 + 0 = 0$.

and $(a+b) \cdot c = 0$ and $a \cdot c + b \cdot c = 0 + 0 = 0$.

40) R is a ring such that $x^2 = x$ for all $x \in R$.

a) Let $x \in R$. $(x+x)^2 = (x+x)(x+x) = x \cdot x + x \cdot x + x \cdot x + x \cdot x$
$= x + \cancel{x} + \cancel{x} + x$

By hypothesis, $(x+x)^2 = x + x$.

Thus $x + x = 0$. Hence $x = -x$.

b) Let $x, y \in R$. By hypothesis $(x+y)^2 = x + y$.

Also $(x+y)^2 = (x+y)(x+y)$
$= x \cdot x + xy + y \cdot x + y \cdot y$
$= x + xy + yx + y = x + y$.

Then $\cancel{x} + y + xy + yx - x - y = 0$

Hence $xy + yx = 0$. Therefore $xy = -yx$

Since $-(yx) \underset{\uparrow \text{by (a)}}{=} -(-yx) = yx$, we obtain $xy = yx$.

Therefore R is a commutative ring.