

3.5. 10) $G = \mathbb{R}$, with + $G' = \mathbb{R}^+ = \{x \in \mathbb{R} \mid x > 0\}$ with \cdot ... $\phi: G \rightarrow G'$ defined by $\phi(x) = 10^x$ for $x \in G$.Claim ϕ is an automorphism.

$$\phi(x) = \phi(y) \Leftrightarrow 10^x = 10^y \Leftrightarrow \log_{10}(10^x) = \log_{10}(10^y)$$

(\log_{10} is a one to one function)

hence ϕ is one to one.

let $r \in G'$. Then $\phi(\log_{10}(r)) = 10^{\log_{10}(r)} = r$ and

$\log_{10}(r) \in G$. Hence ϕ is onto.

$$\phi(x+y) = 10^{x+y} = 10^x \cdot 10^y = \phi(x) \cdot \phi(y).$$

Therefore ϕ is a group homomorphism.Therefore ϕ is a group automorphism.