

Advanced Calculus

M A T H 2 5 1

FIRST MIDTERM

(Duration : 120 mins.)

9th November 1996

[10 + 10 + 10] , [15 + 15] , [20] , [20]

1.

(a) Given $A, B \subseteq \mathbb{R}$ which are bounded from above prove that $A \cup B \subseteq \mathbb{R}$ is bounded from above.

(b) Given $C \subseteq \mathbb{R}$ which is bounded from above and $D \subseteq \mathbb{R}$ which is bounded from below, is it always true that $C \cup D \subseteq \mathbb{R}$ is bounded from above ? Give your reasons.

(c) Given $C, D \subseteq \mathbb{R}$ as in (b) what can you say about $C \cap D \subseteq \mathbb{R}$?

2.

(a) Prove that for any $x \in \mathbb{R}^2$ and $\varepsilon > 0$, the set

$$\overline{B}(x, \varepsilon) = \{y \in \mathbb{R}^2 \mid |y - x| \leq \varepsilon\}$$

is a closed set.

(b) Prove that an open subset of \mathbb{R}^2 is the union of a family of closed set with non-empty interiors.

3.

Prove without using the Heine-Borel Theorem that the set

$$[-1, 1] \times [-1, 1] - \{(0, 0)\}$$

is not compact.

P L E A S E T U R N O V E R !

4.

Let F be an ordered field in which every set which is bounded from below has a supremum. Prove that F is Archimedean.