UNIFORMIZATION, RIEMANN SURFACES AND ARITHMETIC

The subject comprises an active field of mathematics where the interplay between
(i) geometry and topology (coverings, group actions and quotient structures),
(ii) complex analysis (automorphic functions),
(iii) algebra (discontinuous groups, Lie groups)
lead to fundamental results in the geometry of complex manifolds (basically Riemann surfaces), complex analysis and arithmetic (solutions of diophantine problems, properties of zeta functions and problems on transcendence of values of holomorphic functions). The remarkable success of the methods in the one-dimensional case is basically due to the simplicity of geometry and complex analysis in dimension one. This simplicity results in the merge of various aspects of the problems involved, leading to complete answers and fundamental progress. In the higher dimensional cases, as a result of the usual deviation from this unifying behaviour, it is a subtle matter to decide on which aspect of uniformization is relevant to a given problem. In these lectures we will concentrate on the one-dimensional case and we will discuss the higher dimensional versions of uniformization briefly, mainly for the purpose of drawing attention to the subtlety of higher dimensional complex geometry.

Course outline :

1. Introduction : Definition and elementary examples of uniformization.
2. Simply connected Riemann surfaces and uniformization via universal coverings.
3. Basic concepts related to the action of $PSL_2(\mathbb{C})$ on $P^1_{\mathbb{C}}$.
4. Domains in $P^1_{\mathbb{C}}$ and uniformization : Triangle groups, Schottky groups.
5. Quasi-conformal maps applied to uniformization.
6. Simultaneous uniformization.
7. Differential equations and uniformization.
8. Applications to the study of compact Riemann surfaces.
9. Introduction to the uniformization of complex surfaces.
10. Arithmetic applications of uniformization.

References :

Classical material :

1. L. R. Ford, *Automorphic Functions*, Chelsea (1951).

2. J. Lehner, *Discontinuous Groups and Automorphic Functions*, AMS (1961).

3. J. Lehner, *A Short Course in Automorphic Functions*, Holt, Rinehart and Winston (1966).

Neo-classical treatment :

1. L. Bers, Simultaneous uniformization, Bull. AMS 66 (1960), 303-314.

2. L. Bers, Uniformization, moduli and Kleinian groups, Bull. London Math. Soc., 4 (1972), 257-300.

3. L. Bers, On Hilbert's 22nd problem, *Mathematical Developments Arising from Hilbert Problems*, Proc. Symp. Pure Math. v. 28 (1976), 559-610.

4. L. Bers, article in *The Mathematical Heritage of Henri Poincare*, Proc. Symp. Pure Math. v. 39.

5. J-B. Bost, *From Number Theory To Physics*, Springer-Verlag (1992), 64-211.

6. H. Farkas, I. Kra, *Riemann Surfaces*, GTM 71, Springer-Verlag (1980).

Algebro-gometric problems :

1. P. Griffiths, Complex analytic properties of certain Zariski open sets on algebraic varieties, Ann. Math. 94 (1971), 21-51.

Arithmetic Problems :

1. S. Lang, *Introduction To Transcendental Numbers*, Addison-Wesley (1966).

2. G. Shimura, *Arithmetic Theory of Automorphic Functions*, Princeton University Press (1970).

3. P. B. Cohen, J. Wolfart, Modular Eembeddings for some non-arithmetic Fuchsian groups, Acta Arithmetica LVI (1990), 93-109.

4. J. Wolfart, several articles on covering radius, referred to in (3).