

## PRELIMINARY EXAM PROBLEMS

### Differential Equations (PDE), 2004/2

1. Solve the Cauchy problem

$$u_y = u_x^3, \quad u(x, 0) = 2x^{3/2}.$$

2. (a) Verify, formally, that the PDE of the form

$$\left\{ \frac{\partial}{\partial x} \left[ F(x, y) \frac{\partial}{\partial x} \right] + \frac{\partial}{\partial y} \left[ G(x, y) \frac{\partial}{\partial y} \right] \right\} \Phi(x, y) = 0$$

has a solution of the type  $\Phi(x, y) = X(x)Y(y)$ , if  $F(x, y)$  and  $G(x, y)$  are "separable" in the variables, i.e.  $F(x, y) = p(x)f(y)$ ,  $G(x, y) = q(x)w(y)$ . Then write down the system of two ODE's for  $X(x)$  and  $Y(y)$ .

(b) If  $\Phi(0, y) = \Phi(1, y) = 0$  for all  $y$ , verify that the  $x$ -dependence of the problem in Part (a) is equivalent to the system

$$\frac{d}{dx} \left[ p(x) \frac{dX}{dx} \right] + \lambda q(x)X = 0, \quad X(0) = X(1) = 0,$$

where  $p(x)$  and  $q(x)$  are real and positive with continuous derivatives in the interval  $[0, 1]$  and  $\lambda$  is constant.

3. Use Duhamel's principle to solve the IVP

$$u_{tt} - u_{x_1x_1} - u_{x_2x_2} - u_{x_3x_3} = x_1 + x_2 + t,$$

$$u(x_1, x_2, x_3, 0) = u_t(x_1, x_2, x_3, 0) = 0.$$

4. Let  $u$  be a solution of IVP

$$u_t - ku_{xx} = 0, \quad x \in R, \quad t > 0,$$

$$u(x, 0) = f(x).$$

where  $f(x)$  is continuous on  $R$ . Assume that  $u(x, t)$  tends to zero uniformly for  $t > 0$  as  $x \rightarrow \pm\infty$ . Show that  $|u(x, t)| \leq M, x \in R, t > 0$ , if  $|f(x)| \leq M, x \in R$ .