Graduate Preliminary Examination
General Topology
Duration: 3 hours

September 26, 2003

1. Consider a topology on the set $[-1, 1]$ with respect to which $U \subseteq [-1, 1]$ is open if either $0 \notin U$ or $(-1, 1) \subseteq U$

(3 pts.) (a) What are the closed subsets of $[-1, 1]$ which do not contain 0?

(3 pts.) (b) Find the closure of the set $[0, 1)$.

(4 pts.) (c) Find the boundary of the set $[0, 1)$.

2.

(5 pts.) a) Show that each of the following is a basis for a topology on $\mathbb{R}$.

$$\mathcal{B}_1 = \{[a, b)|a < b\}$$

$$\mathcal{B}_2 = \{(a, b]|a < b\}$$

(5 pts.) b) Let τ_1, τ_2 be the topologies generated by $\mathcal{B}_1$ and $\mathcal{B}_2$ respectively. Are $(\mathbb{R}, \tau_1)$ and $(\mathbb{R}, \tau_2)$ homeomorphic? Prove your answer.

3. Consider $X = \{a, b, c\}$ with the topology $\{\{a\}, \{a, b\}, \{a, b, c\}\}$

(2 pts.) a) Is X is Hausdorff? Why?

(6 pts.) b) Show that X is path connected.

(2 pts.) c) Is X is compact? Why?

4. Let X be a topological space. Remember that a family $\mathfrak{h}$ of (any) subsets of X is said to be *locally finite* if each point of X has a neighbourhood that has non-empty intersection with only finitely many elements of $\mathfrak{h}$.

(5 pts.) (a) If X is compact, prove that every locally finite family of subsets of X is finite.

(5 pts.) (b) Let Y be a topological space in which every locally finite family of subsets is finite. Prove that Y is limit point compact (i.e. every infinite subset of Y has an accumulation point).