ABSTRACT

BLACK BOX GROUPS AND RELATED GROUP THEORETIC CONSTRUCTIONS

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The present thesis aims to develop an analogy between the methods for recognizing a black box group and the classification of the finite simple groups. We propose a uniform approach for recognizing simple groups of Lie type which can be viewed as the computational version of the classification of the finite simple groups. Similar to the inductive argument on centralizers of involutions which plays a crucial role in the classification project, our approach is based on a recursive construction of the centralizers of involutions in black box groups. We present an algorithm which constructs a long root $\mathrm{SL}_2(q)$-subgroup in a finite simple group of Lie type of odd characteristic p extended possibly by a p-group. Following this construction, we take the Aschbacher's "Classical Involution Theorem" as a model in the final recognition algorithm and we propose an algorithm which constructs all root $\mathrm{SL}_2(q)$-subgroups corresponding to the nodes in the extended Dynkin diagram, that is, our approach is the construction of the the extended Curtis - Phan - Tits presentation of the finite simple groups of Lie type of odd characteristic which further yields the construction of all subsystem subgroups which can be read from the extended Dynkin diagram. In this thesis, we present this algorithm for the groups $\mathrm{PSL}_n(q)$ and $\mathrm{PSU}_n(q)$. We also present an algorithm which determines whether the p-core (or "unipotent radical") $O_p(G)$ of a black box group G is trivial or not where $G/O_p(G)$ is a finite simple classical group of Lie type of odd characteristic p answering a well-known question of Babai and Shalev.

The algorithms presented in this thesis have been implemented extensively in the computer algebra system GAP.

Keywords: Black Box Groups, Groups Of Lie Type.