

ABSTRACT

REAL LEFSCHETZ FIBRATIONS

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In this thesis, we present real Lefschetz fibrations. We first study real Lefschetz fibrations around a real singular fiber. We obtain a classification of real Lefschetz fibrations around a real singular fiber by a study of monodromy properties of real Lefschetz fibrations. Using this classification, we obtain some invariants, called real Lefschetz chains, of real Lefschetz fibrations which admit only real critical values. We show that in case the fiber genus is greater than 1, the real Lefschetz chains are complete invariants of directed real Lefschetz fibrations with only real critical values. If the genus is 1, we obtain complete invariants by decorating real Lefschetz chains.

For elliptic Lefschetz fibrations we define a combinatorial object which we call necklace diagrams. Using necklace diagrams we obtain a classification of directed elliptic real Lefschetz fibrations which admit a real section and which have only real critical values. We obtain 25 real Lefschetz fibrations which admit a real section and which have 12 critical values all of which are real. We show that among 25 real Lefschetz fibrations, 8 of them are not algebraic. Moreover, using necklace diagrams we show the existence of real elliptic Lefschetz fibrations which can not be written as the fiber sum of two real elliptic Lefschetz fibrations. We define refined necklace diagrams for real elliptic Lefschetz fibrations without a real section and show that

refined necklace diagrams classify real elliptic Lefschetz fibrations which have only real critical values.

Keywords : Lefschetz fibrations, real structure, real Lefschetz fibrations, real Lefschetz chains, necklace diagrams.