PROBLEMS (2)

1. Let X be a topological space.

(A) Prove that for any $A \subseteq X$, the union of A with the set of all its accumulation points is a closed set.

(B) Prove that in order for the set of accumulation points of all subsets of X to be closed it is necessary and sufficient that the accumulation points of the singleton $\{a\}$ is closed for every $a \in X$.

2. Let X be an ordered set. To be precise, there is a relation $\geq$ on X which is reflexive, anti-symmetric, transitive. The *right order topology* on X is the topology which admits as a base the family $\mathcal{R} = \{R_x \ : \ x \in X\}$ where for each $x \in X$

$$R_x = \{y \in X : y \geq x\}$$

(A) Prove that the right order topology is a T_0 topology in which the intersection of an <u>arbitrary</u> family of open sets is open.

(B) Prove that $y \geq x$ iff $x \in \overline{\{y\}}$ with respect to the right order topology on X.

(C) Conversely, given any T_0 topology $\mathcal{J}$ on X in which the intersection of an <u>arbitrary</u> family of open sets is open, prove that the relation ρ on X where $y \rho x$ is defined by $x \in \overline{\{y\}}$, is an order relation with respect to which $\mathcal{J}$ is exactly the right order topology.

3. (A) Is the intersection of two dense subsets in a topological space always dense ?

(B) Let X be a topological space. Prove that the intersection of two open and dense subsets of X is open and dense.

(C) If $\widetilde{\mathfrak{H}}$ is the set of all open and dense subsets in X, prve that $\mathfrak{H} = \widetilde{\mathfrak{H}} \cup \{\emptyset\}$ is a topology on X.

(D) Let $\widetilde{X}$ be the topological space with carrier set X and topology $\mathfrak{H}$. Prove that a function $f : \widetilde{X} \longrightarrow \mathbb{R}$ is continuous iff it is constant.